CELNAV

Celestial navigation in POSIX shell, for the iPad mini

Version 1.1 — self-contained, offline, no almanac to buy and nothing that expires. With a training mode that teaches the subject from first principles.

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1. What this is, and why it is built this way

CELNAV takes sextant altitudes and gives you a fix, with the full working shown and the solution drawn on screen. It is one shell script. It needs no internet, no downloaded almanac, no Python, and no libraries.

Four constraints shaped the design, and each one has a consequence worth knowing.

1. It must work when nothing else does. Consequence: the almanac is *computed*, not looked up. The positions of the Sun, Moon, four planets and 58 stars come from orbital theory built into the script. There is no data file to lose, no table that runs out at the end of the year, and no year in which you need a new edition.

2. It must run on whatever shell an iPad can be persuaded to give you. Consequence: the whole program is POSIX `sh` plus `awk`, the largest common denominator across iSH, a-Shell, Termux, macOS and Linux. Nothing is assumed beyond the shell and an `awk` with trigonometric functions. There is no compiler step and no package to install.

3. It must be one file. Consequence: `celnav` carries its own computation engine inside it as text. The first time you run it, it writes that engine out to `~/.celnav/` and uses it from there. Moving the app to a new device is moving one file.

4. The working must be visible. Consequence: every intermediate value is printed — GHA, declination, LHA, dip, refraction, parallax, semi-diameter, Hc, Zn, intercept. If a fix looks wrong you can see where it went wrong, and you can check any line of it by hand against a paper almanac. The program is a fast, tireless assistant, not a black box.

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2. Installing on the iPad mini

There are two good ways to get a shell on an iPad. **iSH** gives you a full Alpine Linux and a package manager. **a-Shell** is faster and easier on the battery but has a fixed set of commands. CELNAV runs on both.

Follow the route you are using. Run **one block at a time** and check the output before moving on.

> **On an iPad, none of this is needed any more.** `celnav` is a browser app: open it in Safari, add it to the home screen, and it caches itself and runs with the antenna down. There is no shell to install, no `awk`, no package manager and no `gawk`. The routes below are for running the **shell build** at a prompt, which is a thing some people would rather have and which still works.

Route A — iSH (Alpine Linux)

A1. (changes things) Install an `awk` that has trigonometry compiled in. The `awk` inside iSH's BusyBox is sometimes built without the maths library, and CELNAV cannot work without `sin`, `cos` and `atan2`.

```
apk update && apk add gawk
```

A2. (read-only) Confirm the maths works. This must print a number near `0.7854`.

```
gawk 'BEGIN{print atan2(1,1)}'
```

A3. (changes things) Put `celnav` where you can run it, and make it executable. Copy the file into iSH first — in iSH, `mount -t ios . /mnt` lets you reach the Files app, or use whatever transfer you already trust.

```
mkdir -p ~/bin && cp celnav ~/bin/celnav && chmod +x ~/bin/celnav
```

A4. (changes things) Add `~/bin` to your path so you can type `celnav` anywhere.

```
echo 'export PATH="$HOME/bin:$PATH"' >> ~/.profile && . ~/.profile
```

A5. (read-only) Check the installation end to end.

```
celnav doctor
```

You should see `awk maths : OK`, a writable data directory, the UTC clock, and `ALL TESTS PASSED` from the self test.

Route B — a-Shell

a-Shell already includes a suitable `awk`, so there is nothing to install.

B1. (changes things) Bring `celnav` into a-Shell's own document folder — the folder that appears under *On My iPad* in the Files app is the simplest path.

B2. (changes things) Make it executable and confirm where you are.

```
chmod +x celnav && pwd
```

B3. (read-only) Run the environment check.

```
./celnav doctor
```

In a-Shell you will usually run it as `./celnav` from that folder rather than installing it on a path.

Verifying the file arrived intact

(read-only) The script is about 140 kB. A truncated transfer will fail the self test rather than give you quiet nonsense, but you can also check the file directly:

```
cksum celnav
```

Expected for version 1.1: `4175943417 142889 celnav`.

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3. The screen

Starting `celnav` with no arguments gives you the menu. Everything on it also has a command-line form, listed in section 5.

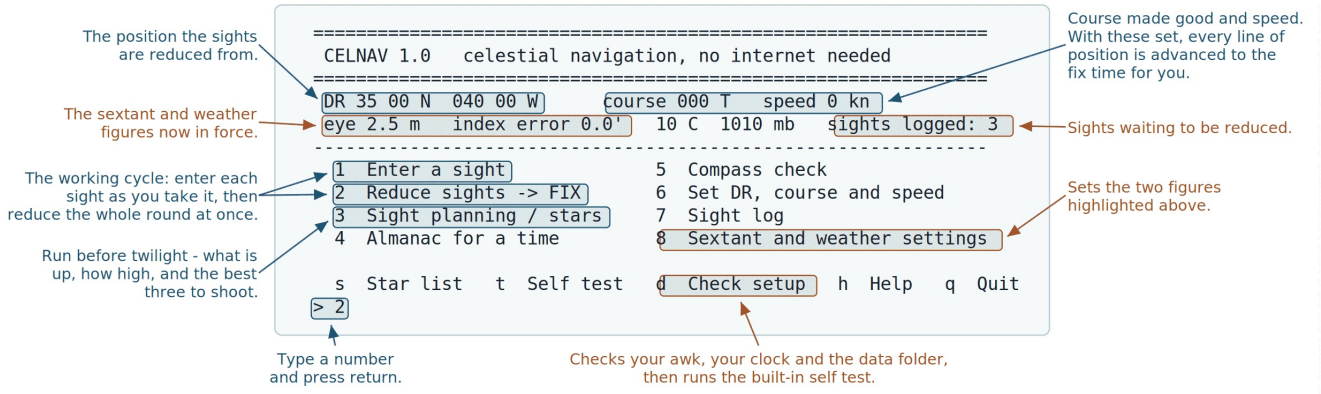


Figure 1 — the main menu. The two status lines at the top always show what the next reduction will be based on, so you can see at a glance whether the DR, the run and the sextant settings are the ones you meant.

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4. The working cycle at sea

Step 1 — tell it where you think you are. Menu item **6**, or `celnav dr "35 10.4 N" "040 20.1 W" 245 11`. The last two numbers are course made good in degrees true and speed over ground in knots. Consequence: with a course and speed set, every line of position is advanced to the fix time automatically, so sights taken an hour apart still give one fix.

Step 2 — plan the round before you go on deck. Menu item **3**, or `celnav plan`. You get twilight times in both UT and local mean time, the moon's phase, every body above five degrees with altitude and azimuth, a sky diagram, and a suggested set of three with the widest azimuth spread. Consequence: you go up with a list, not a hope.

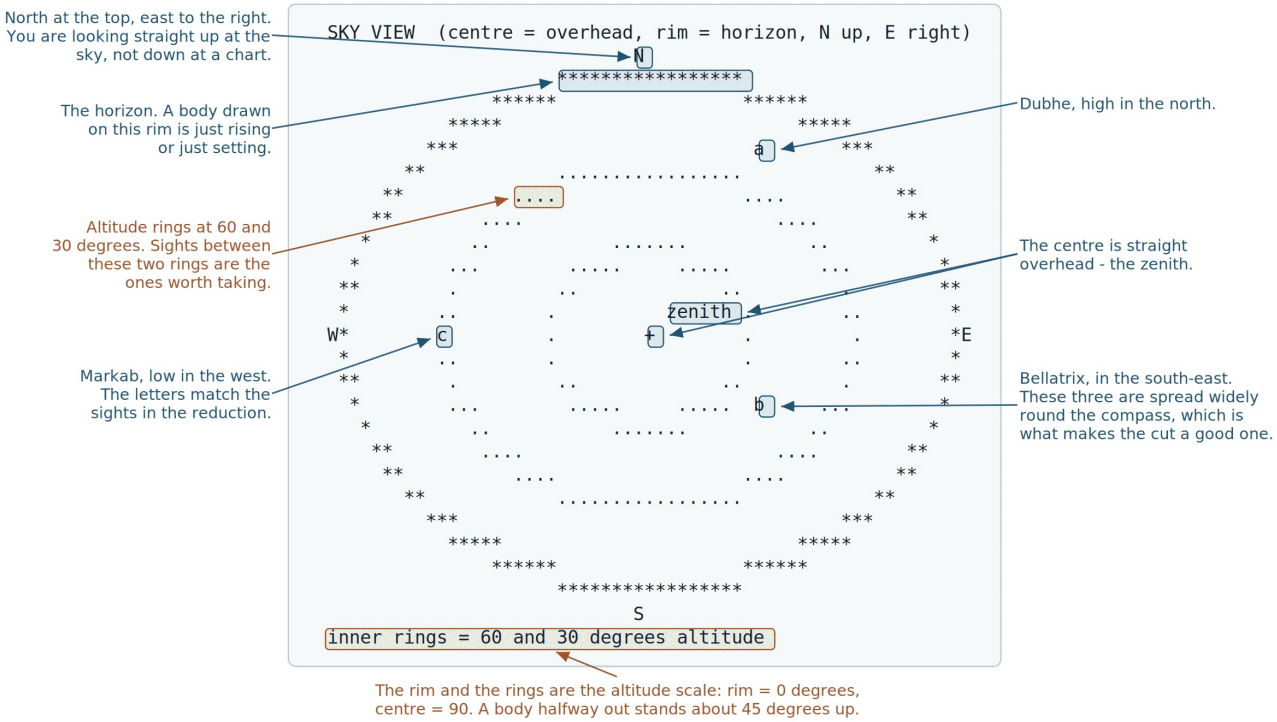


Figure 2 — the sky view from `celnav plan`, and the same diagram after a reduction. It is a picture of the sky above you, not a chart: the point straight overhead is at the centre and the horizon is the outer rim, so a body near the rim is low down and one near the centre is nearly overhead.

Step 3 — shoot, and write down the time to the second. Time is what matters most. Four seconds of clock error is about one mile of longitude. Everything else in the chain is smaller than that.

Step 4 — enter each sight. Menu item **1**, or `celnav sight "2026-08-29 07:30:00" Dubhe C "19 32.1" 1.5 3.0` (UT, body, limb, Hs, index error in minutes, height of eye in metres). Sights accumulate in a log until you clear them.

Step 5 — reduce and plot. Menu item **2**, or `celnav fix`. You get the full working for each sight, the fix, the residuals, a sky view and the intercept plot.

Step 6 — read the geometry line before you believe the fix. CELNAV tells you how far one arcminute of sight error moves the fix, and in which direction. Consequence: a fix from three bodies bunched in one quarter of the sky is reported as weak *before* you steer on it.

Step 7 — clear the log. Menu item **7**, then `c`. Or `celnav clear`.

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5. Command reference

Everything in the menu has a command-line equivalent, so you can script it or get a single answer without navigating menus.

Command	What it does
`celnav`	interactive menu
`celnav plan ["UT"]`	twilight, moon, visible bodies, best three, sky diagram
`celnav alm ["UT"] [bodies]`	GHA, Dec, SD, HP for the bodies you name
`celnav sight "UT" body limb Hs [IE] [eye]`	record one sight
`celnav fix ["UT of fix"]`	reduce the log, compute the fix, draw the plot
`celnav noon [ "YYYY-MM-DD" ] [ Hs ] [ limb ]`	local apparent noon, and latitude from a meridian altitude
`celnav polaris [ "UT" ] [ Hs ]`	latitude from Polaris, any time the horizon shows
`celnav compass "UT" body <bearing> [variation]`	compass error and deviation
`celnav devcard "DATE" body <variation> "hdg,time,brg; ..."`	a swing, as a deviation card
`celnav current "DRlat" "DRlon" "FIXlat" "FIXlon" <hours>`	set and drift
`celnav ep "lat" "lon" <crs> <spd> <hrs> [set] [drift]`	the DR run, and the EP
`celnav table <lat> <dec> [LHA]`	a Pub. 249 page, generated
`celnav stars ["UT"]`	the 57 navigational stars plus Polaris, SHA and Dec
`celnav dr <lat> <lon> [course] [speed]`	set the DR
`celnav log` / `celnav clear`	list or empty the sight log
`celnav doctor`	environment check plus self test
`celnav test`	self test only
`celnav where`	show file locations
`celnav reinstall`	rewrite the engine after replacing the script
`celnav learn`	the training menu
`celnav lesson <code>`	one lesson, e.g. `celnav lesson R3`
`celnav walk`	the annotated walkthrough of a real sight
`celnav drill [type]`	a marked drill; type is corr, alm, red, int, full or fix
`celnav sandbox`	the what-if triangle
`celnav syllabus`	the lesson list with your progress
`celnav day` / `night` / `plain`	colour mode

Angle formats. All of these are understood, anywhere an angle is asked for:

```
35 10.4 N        040 20.1 W        -35.1733        040:20.1W        35d10.4'N
```

Altitudes are degrees and decimal minutes: `32 14.6`. A single number with a decimal point is read as decimal degrees.

Time formats. `2026-08-29 07:30:00` everywhere. In the menu you may type just `073000` or `07:30:00` and today's UT date is assumed.

Limb. `L` lower limb, `U` upper limb, `C` centre — use `C` for stars and planets.

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6. How the numbers are made

This section is the reasoning behind the answers, so that you can check them and so that anyone maintaining the script knows what each part is for.

6.1 Time

1. Your UT date and time become a Julian Date.
2. Greenwich apparent sidereal time follows from it, including nutation, which gives the GHA of Aries directly.
3. Body positions are computed at Terrestrial Time, which is UT plus Delta-T. CELNAV models Delta-T as about 69 seconds in the mid 2020s, rising slowly. Consequence: an error of even 30 seconds in Delta-T moves the Moon by 0.3 arcminutes and everything else by far less, so this is never the weak link.

6.2 The almanac

Different bodies need different treatment, and each choice has a reason.

1. **Stars** come from a J2000 catalogue of the 57 navigational stars plus Polaris, with proper motion, precession, nutation and annual aberration applied to the date. Measured accuracy: better than 0.02 arcminutes.
2. **The Sun and planets** come from Keplerian orbital elements corrected by a fitted series of periodic terms. The raw elements alone leave Saturn several arcminutes out, because of its long-period resonance with Jupiter; the correction series removes that. Consequence: Saturn is now the *most* accurate of the planets rather than the worst.
3. **The Moon** uses the full main-problem lunar series — sixty terms in longitude and distance, sixty in latitude. The Moon earns this because it moves half a degree an hour and has a parallax approaching a degree; a coarse Moon would be the largest error in the whole program.

6.3 Sextant corrections

Applied in this order, and each one printed so you can check it.

1. **Index error** — as you enter it, positive meaning on the arc.
2. **Dip** — 1.76 times the square root of the height of eye in metres, in arcminutes. This gives apparent altitude, `Ha`.
3. **Refraction** — Bennett's formula, scaled for temperature and pressure. Consequence: a real temperature and pressure matter mainly for low sights; above about 25 degrees the whole adjustment is a tenth of a mile.
4. **Parallax in altitude** — horizontal parallax times the cosine of `Ha`. Negligible for the Sun, up to about an hour of arc for the Moon.
5. **Semi-diameter** — added for a lower limb, subtracted for an upper limb. For the Moon it is augmented as the Moon rises, because you are nearer to it when it is overhead.

The result is `Ho`, the observed altitude: what a perfect observer at the centre of the Earth would have measured.

6.4 Sight reduction

No tables, and no assumed position rounded to a whole degree. CELNAV computes the navigational triangle directly at your DR:

```
LHA = GHA + longitude (east positive)
Hc  = asin( sin L sin d + cos L cos d cos LHA )
Zn  = atan2( -cos d sin LHA ,  cos L sin d - sin L cos d cos LHA )
```

The intercept is `(Ho - Hc)` in minutes, which is miles; positive means the true position lies *toward* the body from your DR.

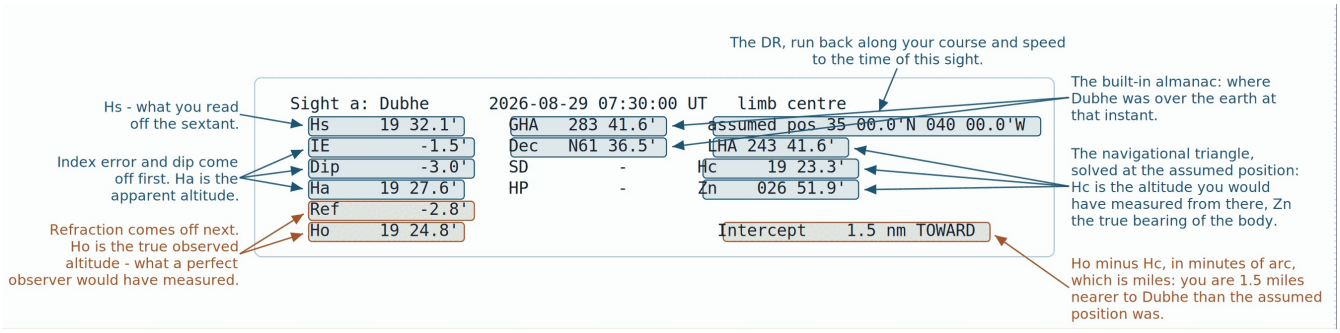


Figure 3 — the working printed for one sight, from Hs on the sextant to the intercept. The left column is the sextant chain, the middle column is the almanac, and the right column is the triangle solved at the assumed position.

Consequence of computing rather than tabulating: your DR is used as it stands, the intercepts stay small, and the answer does not depend on which assumed position you happened to pick.

6.5 The fix

Each line of position says that the offset of the true position from the DR, in miles north and east, satisfies

```
dN cos Zn + dE sin Zn = intercept
```

With two or more sights this is solved by least squares, so every sight contributes and none is privileged. Then:

1. The answer becomes the new assumed position, and the whole reduction runs again.
2. That repeats until the position stops moving.

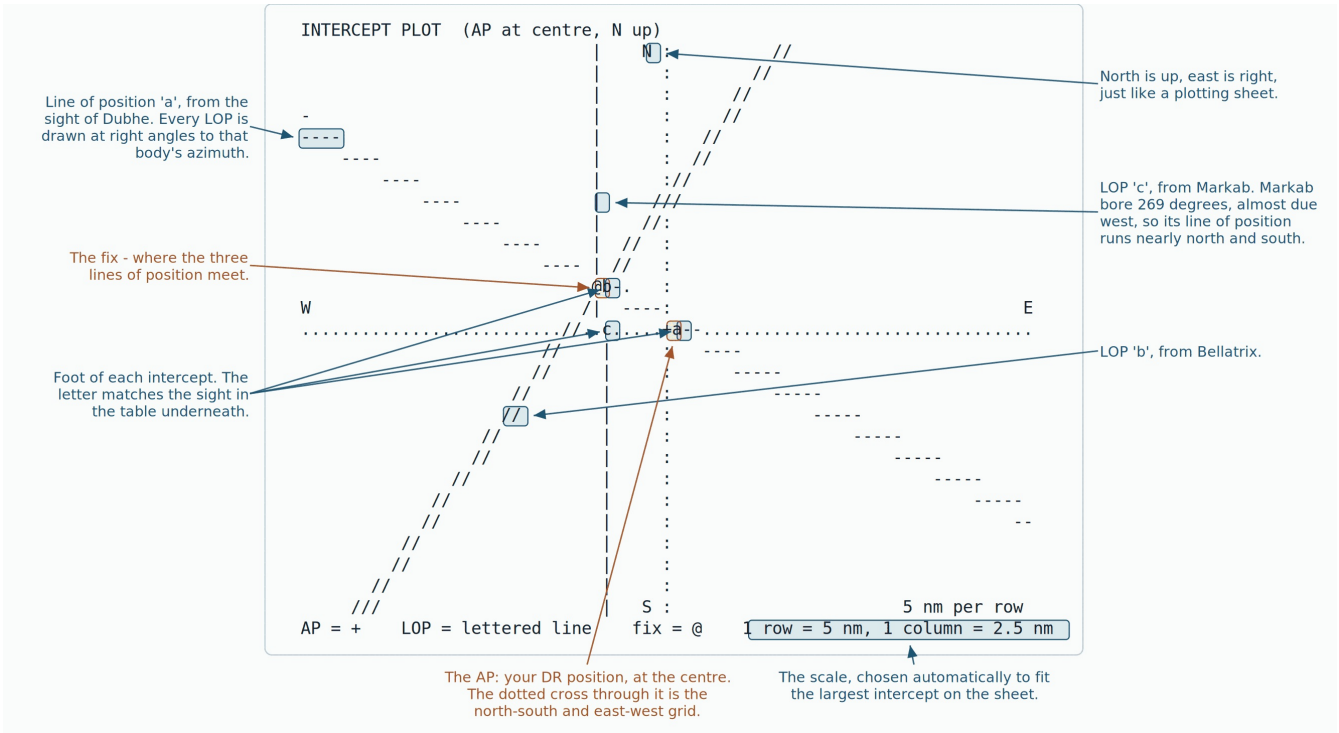


Figure 4 — the intercept plot. It is a plotting sheet drawn in characters: your DR at the centre, one lettered line for each sight, and the fix where they cross. The letters match the summary table printed underneath it.

Consequence: the known weakness of the intercept method — that it is a straight-line approximation to a circle, and drifts when the DR is far out or the body is high — is removed. Tested against exact synthetic sights worldwide, the converged fix lands within 0.11 miles of truth even from a DR 25 miles out.

6.6 Running fix

For each sight, the assumed position is your DR at fix time *run backwards* along your course and speed to the time of that sight. Because the vessel and the assumed position move together, the offset of the fix from the DR is the same quantity for every sight, and they combine directly.

Consequence: sun-run-sun, a morning sun line crossed with a noon latitude, or a spread of star sights taken over twenty minutes at speed — all work with no separate “advance the LOP” step on your part. Set course and speed and it is done.

6.7 Fix geometry

From the same normal equations that give the fix, CELNAV computes the error ellipse for one arcminute of sight error, and reports its long axis and the bearing of that axis.

- Under about 1.6 miles: a sound cut.
- 1.6 to 3 miles: fair; a body more nearly at right angles would tighten it.
- Over 3 miles: reported as a weak cut, with the advice to add a body 40 to 120 degrees away in azimuth.
- Parallel lines of position: refused outright, with an explanation.

Consequence: you are told how far to trust the fix, in miles, at the moment you get it.

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7. Learning and practising

Menu item **9**, or `celnav learn`. It is built on the same engine as the rest of the program, so a lesson and the working tool can never drift apart — the diagrams are drawn by the same code that draws your fix, and the drill problems come from the same almanac.

There are two tracks. The **teaching track** starts from nothing and builds up; the **practice track** gives you problems and marks them. You can use either without the other.

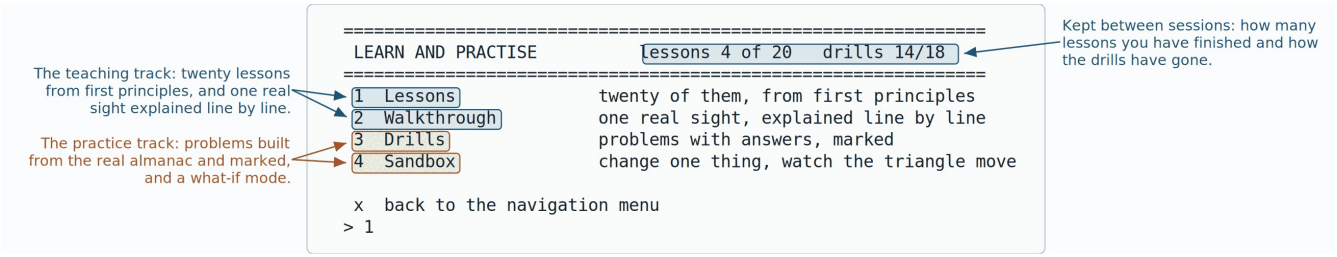


Figure 5 — the training menu. Progress is kept between sessions, so you can do a lesson at a time and pick up where you left off.

7.1 Lessons

Thirty-three lessons in nine modules: a preliminary module on dead reckoning, the three kinds of position and set and drift — offered with a three-question gate, so anyone who already works that way can skip it — then four modules of five, from first principles to the fix, then choosing bodies and checking the compass, latitude taken directly — at noon from the sun and all night from Polaris — and the bodies that are not the sun or a star: the moon, where the corrections stop being bookkeeping, and the planets, where they almost vanish — and last the tables themselves: what Pub. 249 is, why Volume 1 needs no interpolation, how the d column works, and an honest comparison with the calculator, with a tables mode that works a sight alongside you and names which of the four classic mistakes you made. Each one is a diagram, a short explanation in numbered steps, and a question to check that it landed. Answer the question correctly and the lesson is ticked off.

```
FOUNDATIONS            what a sight measures, the geographical position,
                       the circle of position, the fix, and why we draw
                       a straight line instead of a circle
TIME AND THE ALMANAC   UT and the cost of clock error, GHA and declination,
                       Aries and SHA, LHA
SEXTANT                taking a sight, index error and dip, refraction,
                       semi-diameter and parallax, and the size of each
REDUCTION AND THE FIX  the assumed position and the triangle, Hc and Zn,
                       the intercept, plotting, crossing and running
```

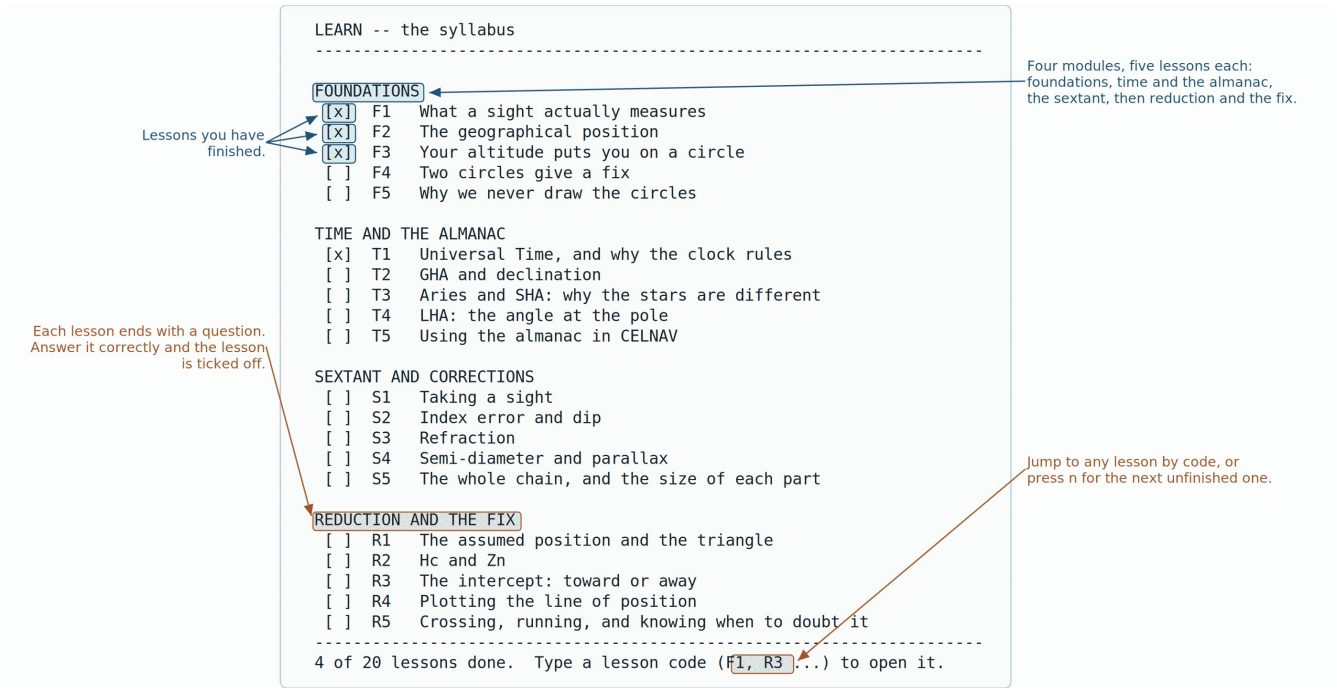


Figure 6 — the syllabus. Type a code to open any lesson, or press `n` for the next one you have not finished.

7.2 The walkthrough

`celnav walk` takes one real sight — the Dubhe sight from the appendix — and goes through it in ten steps, computing each value as it explains why that line exists. It is the fastest way to see the whole method end to end, and it is the same arithmetic the program runs on your own sights.

7.3 Drills

`celnav drill` generates problems from the real almanac, with a different one every time, and marks your answer:

Drill	You are given	You work out
corrections	Hs, index error, height of eye, temperature, pressure	Ho
almanac	a body and a time	GHA and declination
reduction	assumed position, GHA, declination	LHA, Hc and Zn
intercept	Ho and Hc	miles, toward or away
complete sight	everything, from the sextant reading	the intercept and azimuth
three-star fix	three azimuths and intercepts	the position

Tolerances are 0.3 minutes on an altitude, half a degree on an azimuth, and two miles on a fix. Right or wrong, the full working is printed afterwards, so a mistake tells you exactly where you went astray. Your running score is kept.

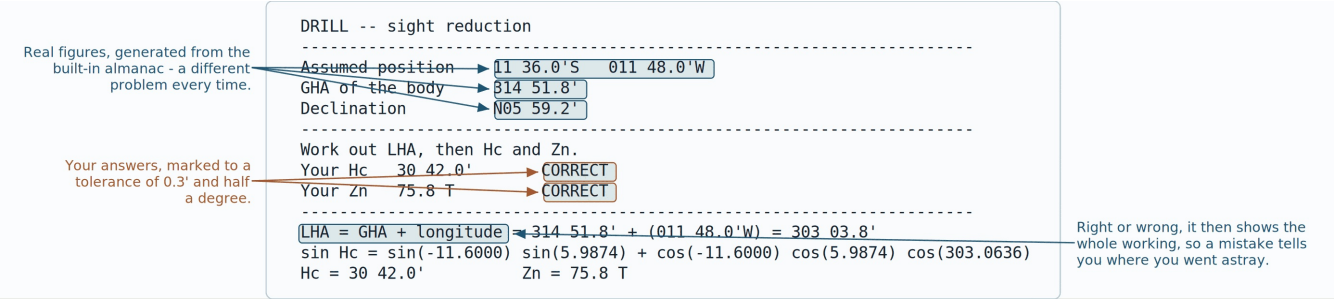


Figure 7 — a reduction drill, answered and marked.

7.4 The sandbox

`celnav sandbox` draws the navigational triangle on your own sky and lets you change one thing at a time — your latitude, the declination, or the LHA — and watch the shape move and Hc and Zn change with it. Set LHA to zero and you are looking at a noon sight; set the declination equal to your latitude and the body passes through your zenith.

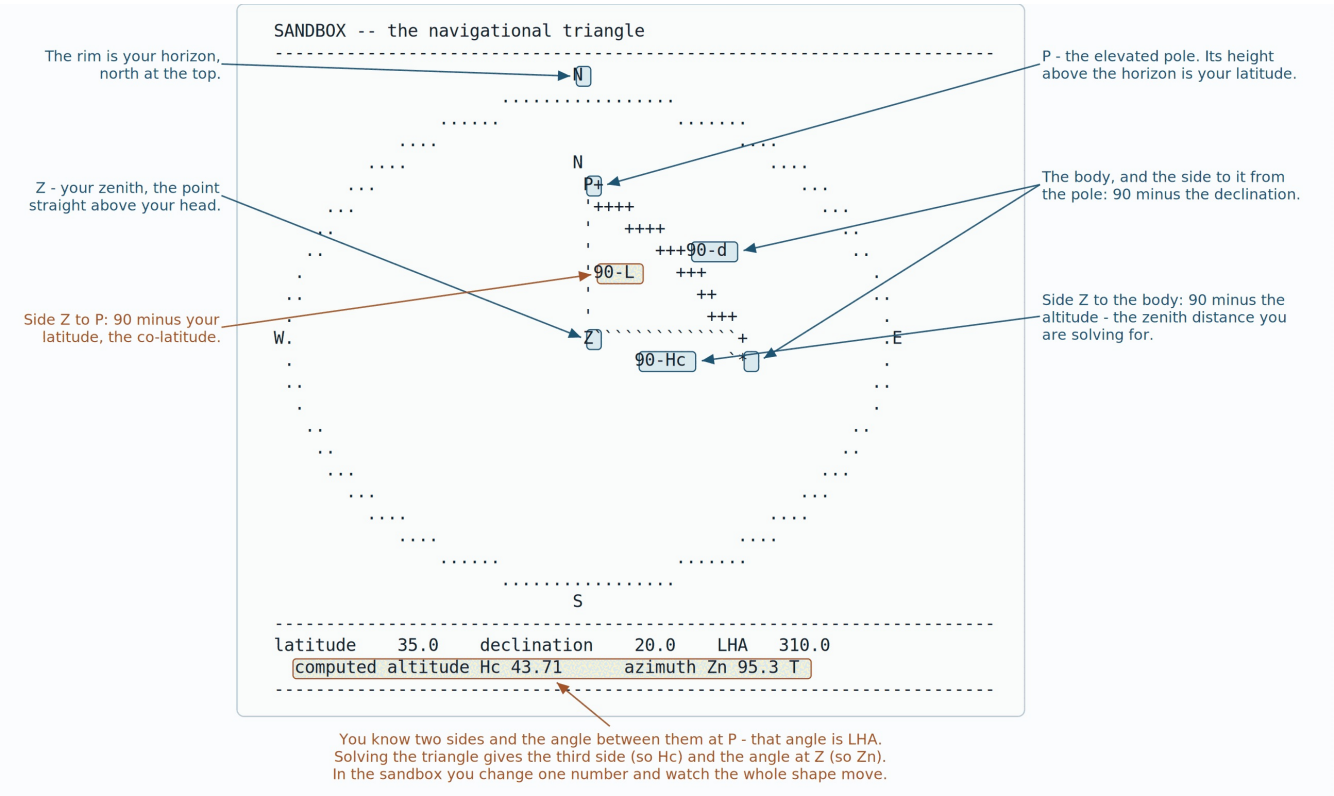


Figure 8 — the sandbox. This is the same triangle the reduction solves for every sight you take.

8. Colour and night vision

Three modes, set from the menu with `c`, or with `celnav day`, `celnav night`, `celnav plain`. The choice is remembered.

- **day** — black background, white text. The assumed position, the fix and the recommended bodies are picked out in green, and the matching symbol in each key is drawn in the same colour, so the legend reads as a colour match

rather than a list.

- **night** — black background, deep red text, with the same highlights in bright red instead of green. There is deliberately no green in night mode: it takes about twenty minutes to get your night vision back, and a white or green screen at 0300 costs you every one of them. Red light at low brightness costs you almost nothing.
- **plain** — no escape sequences at all. Use it when piping output to a file or a printer, and note that CELNAV switches to it automatically whenever its output is not a terminal, so a redirected fix is always clean text.

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9. Accuracy — what was actually measured

The almanac was checked against an independent high-precision ephemeris over 1990–2076, sampling every 11 days. Errors are the total angular difference in apparent position, in arcminutes.

Body	RMS error	Worst case	Notes
Navigational stars	0.005′	0.011′	all 58 checked
Sun	0.06′	0.22′	
Moon	0.11′	0.33′	measured over 2024–2036
Venus	0.09′	0.42′	worst near inferior conjunction, when it is unobservable
Mars	0.06′	0.35′	worst near opposition
Jupiter	0.03′	0.06′	
Saturn	0.01′	0.03′	

What this means in practice. One arcminute is one mile. The almanac contributes well under half a mile everywhere, and under a tenth of a mile for stars. A good sextant observation from a small vessel is worth one to two miles. The almanac is therefore not your limiting factor, and will not become one.

The fix arithmetic was checked separately by generating exact sights for 200 random positions worldwide — all latitudes to 80 degrees, all longitudes, 2026 to 2033, running fixes at up to 14 knots — reducing them from a DR up to 25 miles away, and comparing the answer to the known truth. Every case with sound geometry came back within **0.11 miles**. The only cases that did not were ones where the lines of position were nearly parallel, and CELNAV reports those as weak cuts.

The self test travels with the app. `celnav test` re-checks fourteen reference positions and the reduction arithmetic against values embedded in the script. It runs offline in about a second and tells you the worst error it found. Run it whenever you doubt the answers.

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10. Limits and cautions

1. **Validity window.** The planetary correction series was fitted over 1985–2080 and is accurate throughout. Outside that span accuracy degrades gracefully but is not guaranteed. Stars, Sun and Moon remain good well beyond it.
2. **Your clock is the weak link.** CELNAV cannot know that your watch is wrong. Set it against a known source and note the rate. Four seconds is a mile.
3. **Low sights.** Below about 15 degrees, refraction becomes uncertain because it depends on the real temperature profile over the sea rather than on the standard model. CELNAV flags these in the planning list. Prefer bodies above 15 degrees when you have the choice.
4. **Very high sights.** Above about 75 degrees the azimuth changes quickly with position and the line of position is noticeably curved. The iterated fix handles this correctly, but the *plot* is a straight-line drawing, so in that case trust the printed fix over the picture.
5. **This is a check on your own work, not a replacement for it.** Carry the tables and know how to use them. A single script on a single device is a single point of failure; the reason the working is printed in full is so that you can continue by hand from any line of it.

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11. Files and formats

Path	What it is
`~/.celnav/engine-1.1.awk`	the computation engine, written out on first run
`~/.celnav/teach-1.1.awk`	the training module
`~/.celnav/progress`	which lessons you have done and how the drills have gone
`~/.celnav/sights.txt`	the sight log
`~/.celnav/celnav.conf`	DR, course, speed, height of eye, index error, temperature, pressure, colour mode

Set `CELNAV_HOME` to put these elsewhere. Set `CELNAV_AWK` to force a particular `awk`, for example `CELNAV_AWK=gawk celnav`.

Sight log format — one sight per line, fields separated by `|`, `#` for comments. You can edit it in any text editor:

```
UT                  |body     |limb|Hs      |IE |eye|temp|press|label
2026-08-29 07:30:00 |Dubhe    |C   |19 32.1 |1.5|3.0|18  |1013 |morning stars
```

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12. Troubleshooting

“no awk with trigonometric functions was found” Your `awk` was built without the maths library. On iSH: `apk add gawk`. On Termux: `pkg install gawk`. Then run `celnav doctor` again.

The self test fails. The most likely cause is a truncated file transfer. Check `cksum celnav` against the value in section 2 and re-copy the file if it differs.

“Single line of position only”. You have one sight, or several on the same azimuth. Add a body well away in azimuth.

The fix is a long way from the DR. That is fine — the iterated reduction is exact regardless of distance. But read the residuals: if they are large, one sight is wrong. Check the *time* of each sight first. A transposed digit in the seconds is the commonest error, and it moves only the longitude.

The plot looks cluttered. It is scaled automatically to fit the largest intercept, so if one sight has a very large intercept the others are squeezed together — which is usually the plot telling you that one sight is bad.

Appendix — a worked example

Three star sights at morning twilight in the North Atlantic, from a DR of 35 00.0’N 040 00.0’W, no way on.

```
celnav dr "35 00 N" "040 00 W" 0 0
celnav sight "2026-08-29 07:30:00" Dubhe     C "19 32.1" 1.5 3.0
celnav sight "2026-08-29 07:34:00" Bellatrix C "49 37.2" 1.5 3.0
celnav sight "2026-08-29 07:38:00" Markab    C "29 01.3" 1.5 3.0
celnav fix
```

CELNAV prints the full working for each sight, then:

```
  FIX   35 09.9'N   040 20.0'W
  Offset from DR: 9.9 nm N, 16.4 nm W   (19.2 nm on 301 T)

  LOP residuals at the fix (nm):
    a Dubhe      Zn 027 T   intercept from DR    1.5   residual  +0.01
    b Bellatrix  Zn 128 T   intercept from DR  -19.0   residual  +0.01
    c Markab     Zn 269 T   intercept from DR   16.2   residual  +0.01
    RMS scatter 0.01 nm  -  this is the quality of your sights

  GEOMETRY: 1.0' of sight error puts the fix out by 0.9 nm along 185 T
            and 0.7 nm across it.  (3 LOPs)
```

The sights here were generated exactly, so the residuals are effectively zero; with real sights the RMS scatter is your own observing error, and it is the honest measure of how good the round was.

Then the sky view — where the three bodies were when you shot them:

```
SKY VIEW  (centre = overhead, rim = horizon, N up, E right)
                          N
                    *****************
                ******                 ******
             *****                         *****
          ***                           a       ***
        **              .................            **
      **          ....                 ....          **
     **        ....                        ....        **
    *         ..          .......            ..         *
   *         ...        .....     .....        ...        *
  **         .          ..           ..          .        **
  *         ..         .          zenith .         ..        *
 W*         c          .        +         .         .        *E
  *         ..         .                  .         ..        *
  **         .          ..              ..          .        **
   *         ...        .....     .....  b      ...        *
    *          ..          .......            ..         *
     **        ....                        ....        **
      **          ....                 ....          **
        **              .................            **
          ***                           ***
            *****                         *****
                ******                 ******
                    *****************
                          S
inner rings = 60 and 30 degrees altitude
```

and the intercept plot — the plotting sheet, drawn in characters:

```
INTERCEPT PLOT  (AP at centre, N up)
                              |    N :          //
                              |      :        //
                              |      :      //
-                             |      :     //
----                          |      :   //
    ----                      |      :  //
        ----                  |      ://
            ----              |     ///
                ----          |    //:
                    ----      |  //  :
                        ---- | //   :
                             -@b-.   :
W                            /|  ----:                                  E
.........................//..c.....+a--................................
                        //    |     :   ----
                       //     |     :      -----
                     //       |     :          -----
                    //        |     :              -----
                  //          |     :                  -----
                 //           |     :                      -----
               //             |     :                          -----
              //               |     :                              --
            //                 |     :
           //                  |     :
         //                    |     :
       ///                     |   S :                        5 nm per row
AP = +    LOP = lettered line    fix = @    1 row = 5 nm, 1 column = 2.5 nm

    body         Zn       intercept  LOP
a   Dubhe        027 T       1.5 nm  toward Ho 19 24.8'  Hc 19 23.3'
b   Bellatrix    128 T      19.0 nm  away   Ho 49 31.8'  Hc 49 50.8'
c   Markab       269 T      16.2 nm  toward Ho 28 55.0'  Hc 28 38.8'
```

Read it like a plotting sheet. The `+` at the centre is your DR. Each lettered line is one line of position, drawn at right angles to that body's azimuth, and the letter sits at the foot of its intercept. The `@` is the fix. Here it lies up and to the left of the DR — about 10 miles north and 16 miles west — which is what the figures above the plot say in words.

The same three sights taken while making 9 knots on 060 T, from a DR of 35 05.0'N 040 15.0'W, are reduced the same way once the run is set — each line of position is advanced to the fix time for you, and the answer comes back on the same position.